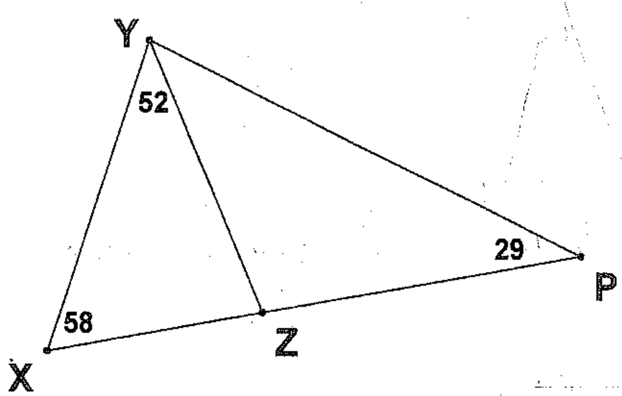
The Bronx High School of Science
Jean Donahue, PhD, Principal
Course: MGS21H – Honors Geometry

Mathematics Department
Vikram Arora, A.P of Mathematics
Instructor: Mr. K. Cheung

Super Trio Maximus: Some Triangle Problems…

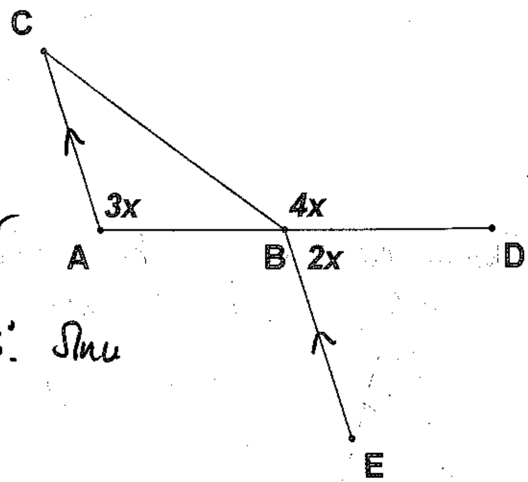
1) In the diagram below, compute $m\angle ZYP$.



Note that in $\triangle XYP$, $m\angle X + m\angle XYP + m\angle P = 180$.
Also note that $m\angle XYP = m\angle XYZ + m\angle PYZ$ by Partition Postulate.
So:

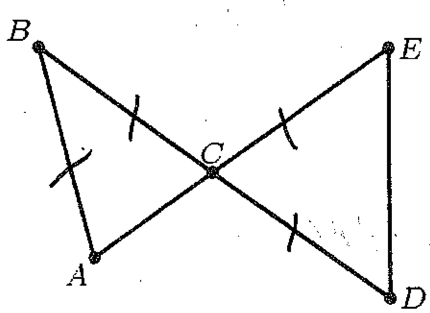
$$m\angle X + m\angle XYZ + m\angle ZYP + m\angle P = 180^\circ.$$
$$58 + 52 + m\angle ZYP + 29 = 180.$$
$$m\angle ZYP + 139 = 180^\circ$$
$$\boxed{m\angle ZYP = 41^\circ.}$$

2) In the diagram at right, $\overline{CA} \parallel \overline{BE}$. Compute $m\angle C$.



Note that $\angle CAB$ and $\angle EBA$ are alternate interior angles. Since $\overline{CA}$ and $\overline{BE}$ are parallel with $\overline{AD}$ as a transversal, the two angles are congruent. Thus, $m\angle EBA = 2x$, so based on the linear pair $\angle ABE$ and $\angle DBE$, $3x + 2x = 5x = 180$, so $x = 36$.
So, $m\angle CBD = 4 \cdot 36 = 144^\circ$, and $m\angle CBA = 180 - 144 = 36^\circ$. Since $m\angle A = 3 \cdot 36 = 108^\circ$, from $\triangle CBA$ we have

$$m\angle C = 180 - (36 + 108) = 180 - 144 = \boxed{36^\circ.}$$



3) (2009 AMC 10B #9) Let $\overline{AE}$ and $\overline{BD}$ intersect at C, with $AB = BC = CD = CE$. If $m\angle A = \frac{5}{2} m\angle B$, compute $m\angle D$.

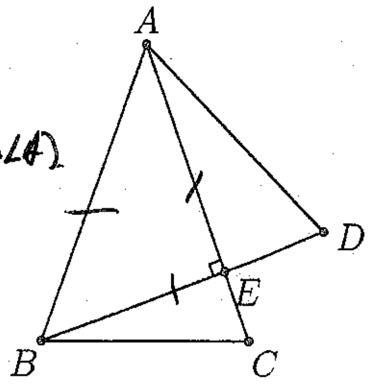
Let $m\angle B = x$. Thus $m\angle A = m\angle BCA = \frac{5}{2}x$. So, from $\triangle ABC$, we have:

$$\frac{5}{2}x + \frac{5}{2}x + x = 180$$
$$5x + x = 180$$
$$6x = 180$$
$$x = 30.$$

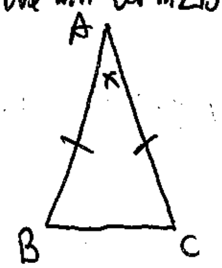
Thus $m\angle BCA = \frac{5}{2} \cdot 30 = 75^\circ$. So, $m\angle ECD = 75^\circ$ as well since $\angle BCA$ and $\angle ECD$ are vertical angles.
So, since $m\angle E = m\angle D$, $m\angle D = \frac{1}{2}(180 - m\angle ECD)$

$$= \frac{1}{2}(180 - 75) = \frac{1}{2} \cdot 105 = \boxed{\frac{105^\circ}{2} = 52.5^\circ.}$$

4) (1996 AHSME #21) In the diagram at right, ΔABC and ΔABD are isosceles triangles with $AB = AC = BD$. $\overline{BD}$ intersects $\overline{AC}$ at E such that $\overline{BD} \perp \overline{AC}$. Compute $m\angle C + m\angle D$.

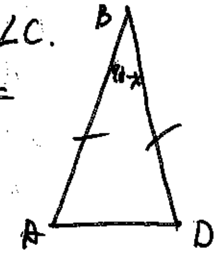


We will let $m\angle BAC = x$. Looking at ΔABC, we see that $m\angle ABC = \frac{1}{2}(180 - m\angle A)$.



Thus, $m\angle ABC = \frac{1}{2}(180 - x) = 90 - \frac{x}{2} = m\angle C$.

In ΔABE, $m\angle AEB = 90°$, so $m\angle ABE = 180 - 90 - m\angle A = \underline{90 - x}$.



If we look at ΔABD, $m\angle D = \frac{1}{2}(180 - m\angle ABD)$, so $m\angle D = \frac{1}{2}(180 - (90 - x)) = \frac{1}{2}(90 + x) = 45 + \frac{x}{2}$.

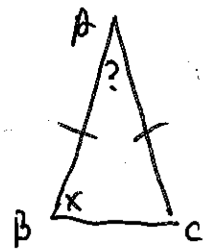
So, we sum:

$$m\angle C + m\angle D = 90 - \frac{x}{2} + 45 + \frac{x}{2} = 90 + 45 \cancel{- \frac{x}{2}} \cancel{+ \frac{x}{2}} = \boxed{135°}.$$

5) (2015 HMNT Team #1) Let ΔABC be isosceles, and let $m\angle ABC = x$. The sum of all possible values of $m\angle BAC$ is $240°$. Compute x.

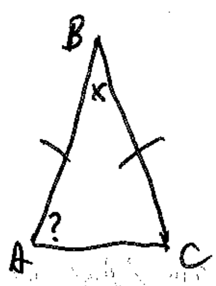
There are three cases to consider, based on which angle is the vertex.

I:



$\angle A$ is the vertex. Thus, $m\angle B = m\angle C = x$, so $m\angle BAC = 180 - 2x$.

II:



$\angle B$ is the vertex. Thus, $m\angle A = m\angle C$, and $m\angle BAC = \frac{1}{2}(180 - x)$
$= 90 - \frac{x}{2}$.

III:

$\angle C$ is the vertex angle. Thus, $m\angle A = m\angle B$, so $m\angle BAC = x$.

These sum to 240, so:

$$240 = 180 - 2x + 90 - \frac{x}{2} + x$$
$$480 = 360 - 4x + 180 - x + 2x$$
$$480 = 540 - 3x$$
$$-60 = -3x$$
$$\boxed{x = 20°}$$