The Bronx High School of Science
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Course: M4(H) – (Honors) Geometry

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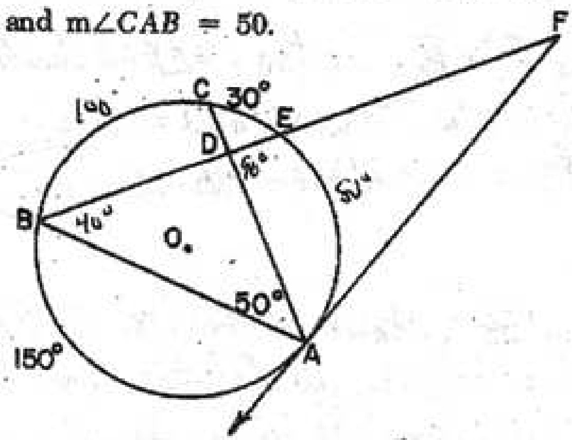
Regents Circle Problems

Sometimes, you'll have to put everything together in order to solve a larger circle problem. A few hints:

- Most of the time, you will not need to draw in additional lines, but it may help.
- The angles are listed in that order for a reason. Remember that you can use an answer from a previous part to help you with a later part.
- Always write out the formulas before you substitute in numbers.
- Check off in the givens what you have used—if you are stuck, go back and look for pieces you haven't used yet.

Source: June 1984 Course III Regents

38 In circle O, $\overrightarrow{FA}$ is a tangent, $\overline{FEB}$ is a secant, $\overline{AC}$ and $\overline{AB}$ are chords, $m\overset{\frown}{CE} = 30$, $m\overset{\frown}{AB} = 150$, and $m\angle CAB = 50$.



Find:

a	$m\overset{\frown}{BC}$	[2]
b	$m\angle EBA$	[2]
c	$m\angle ADE$	[2]
d	$m\angle F$	[2]
e	$m\angle FAC$	[2]

a) $m\angle BAC = \frac{1}{2} m\overset{\frown}{BC}$

$50 = \frac{1}{2} \cdot m\overset{\frown}{BC}$

$\boxed{100 = m\overset{\frown}{BC}}$

b) $m\angle EBA = \frac{1}{2} m\overset{\frown}{EA}$.

$m\overset{\frown}{EA} = 360 - m\overset{\frown}{AB} - m\overset{\frown}{BC} - m\overset{\frown}{CE}$

$= 360 - 150 - 100 - 30 = 80°$.

$m\angle EBA = \frac{1}{2} \cdot 80° = \boxed{40°}$

c) $m\angle ADE = \frac{1}{2}(m\overset{\frown}{BC} + m\overset{\frown}{AE})$

$= \frac{1}{2}(100+80) = \frac{1}{2}(150) = \boxed{90°}$

d) $m\angle F = \frac{1}{2}(m\overset{\frown}{AB} - m\overset{\frown}{AE})$

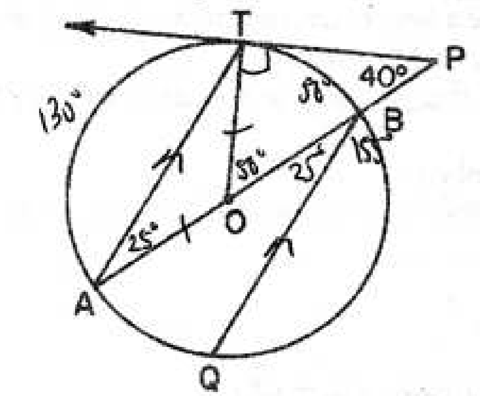
$= \frac{1}{2}(150-80) = \frac{1}{2} \cdot 70 = \boxed{35°}$

e) $m\angle FAC = \frac{1}{2}(m\overset{\frown}{AC})$

$= \frac{1}{2}(110) = \boxed{55°}$

Source: August 1987 Course III Regents

42 In the accompanying diagram of circle O, $\overline{PBOA}$ is a secant, $\overrightarrow{PT}$ is tangent to circle O at T, $m\angle P = 40$, and $\overline{QB} \parallel \overline{AT}$.



Find:

a $m\angle BOT$ [2]
b $m\angle A$ [2]
c $m\widehat{AT}$ [2]
d $m\angle ATO$ [2]
e $m\angle PBQ$ [2]

a) Note $\overline{TP} \perp \overline{TO}$, so $m\angle OTP = 90^\circ$.
Thus, $m\angle BOT = 180 - m\angle P - m\angle OTP$
$= 180 - 40 - 90 = \boxed{50^\circ}$

b) $m\angle A = \frac{1}{2} m\widehat{BT}$
$= \frac{1}{2} \cdot 50^\circ = \boxed{25^\circ}$

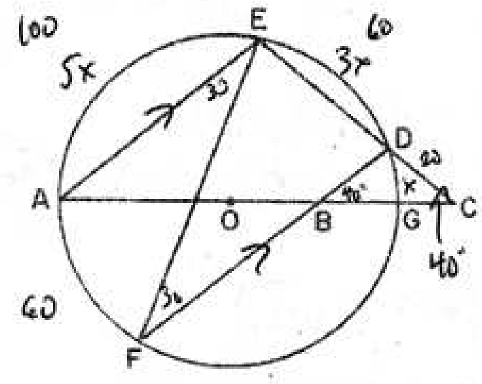
c) $m\widehat{AT} = 180 - m\widehat{TB}$ — note $\widehat{ATB}$ is a semicircle.
$= 180 - 50$
$= \boxed{130^\circ}$

d) $m\angle ATO = m\angle TAO$ since $\triangle ATO$ is isosceles. Thus $m\angle TAO = m\angle ATO = \boxed{25^\circ}$

e) Since $\overline{AT} \parallel \overline{BQ}$, $m\angle TAO = m\angle ABQ$ since they are alt. interior angles. Thus $m\angle ABQ = 25^\circ$, and
$m\angle PBQ = 180 - m\angle ABQ = 180 - 25 = \boxed{155^\circ}$

Source: January 1989 Course III Regents

37 In the accompanying diagram of circle O, $\overline{AE}$ and $\overline{FD}$ are chords, $\overline{AOBG}$ is a diameter and is extended to C, $\overline{CDE}$ is a secant, $\overline{AE} \parallel \overline{FD}$, and $m\widehat{AE}:m\widehat{ED}:m\widehat{DG} = 5:3:1$.



Find:

a $m\widehat{DG}$ [2]
b $m\angle AEF$ [2]
c $m\angle DBG$ [2]
d $m\angle DCA$ [2]
e $m\angle CDF$ [2]

a) Since $\overline{AG}$ is a diameter, $m\widehat{AEG} = 180^\circ$. So, let $m\widehat{DG} = x$, $m\widehat{ED} = 3x$, and $m\widehat{AE} = 5x$. Thus:
$x + 3x + 5x = 180 \rightarrow 9x = 180 \rightarrow x = 20$.
So $\boxed{m\widehat{DG} = 20^\circ}$

b) Since $\overline{AE} \parallel \overline{FD}$, $m\angle AEF = m\angle EFD$. $m\angle EFD = \frac{1}{2} m\widehat{ED}$,
so $m\angle EFD = m\angle AEF = \frac{1}{2}(3 \cdot 20) = \frac{1}{2} \cdot 60 = \boxed{30^\circ}$

c) $m\angle DBG = \frac{1}{2}(m\widehat{DG} + m\widehat{AF})$
$= \frac{1}{2}(20 + 60) = \frac{1}{2} \cdot 80 = \boxed{40^\circ}$

d) $m\angle DCA = \frac{1}{2}(m\widehat{AE} - m\widehat{DG})$
$= \frac{1}{2}(100 - 20) = \frac{1}{2} \cdot 80 = \boxed{40^\circ}$

e) $m\angle CDF = 180 - m\angle C - m\angle DBG$ ($\triangle CDB$)
$= 180 - 40 - 40$
$= 180 - 80 = \boxed{100^\circ}$