The Bronx High School of Science
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Course: MGS21(H) – (Honors) Geometry

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And Never the Two Shall Meet—Parallel Lines Review

1) **(Normal)** Compute the number of sides that a regular *n*-gon should have so that each interior angle measures $170°$. What is the numerical value of the sum of the measures of all of the interior angles of this polygon? We have two ways of thinking about this:

$\frac{180(n-2)}{n} = 170°$

$180(n-2) = 170n$

$180n - 360 = 170n$

$10n = 360$

$\boxed{n = 36.}$

* Here, we consider each of the interior angles of the regular n-gon, and set it equal to 170°.

OR

$180 - 170 = 10°$

$\frac{360}{10°} = \boxed{36}$

Here, since an exterior angle is supplementary to an interior angle, each exterior angle has a measure of 10°. We thus divide to calculate the number of sides.

Either way, we get $n = 36$.

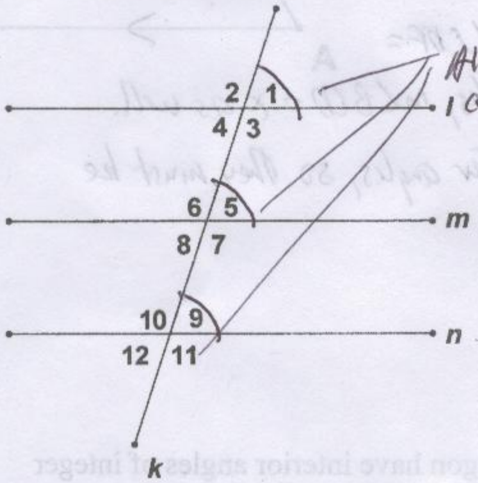
The interior angles of a 36-gon thus sum to:

$180(36-2)$

$= 180(34)$

$\boxed{= 6120°.}$

2) **(Normal)** Note that we have used the fact that the Transitive Property works for parallel lines. However, we've never actually proved that—which needs to be rectified. Prove that Transitive Property works for parallel lines—that is, if two lines are each parallel to a third line, then the lines are parallel. Use the diagram below to assist you. List appropriate givens and a prove statement.



Your proof may vary slightly.

All corresponding.

Statements	Reasons
1. $l \parallel m$	1. Given.
2. $\angle 1 \cong \angle 5$	2. If two parallel lines are cut by a transversal, then corresponding angles are congruent.
3. $m \parallel n$	3. Given.
4. $\angle 5 \cong \angle 9$	4. Same as step 2
5. $\angle 1 \cong \angle 9$	5. Transitive Property of Congruence.
6. $l \parallel n$	6. If two lines are cut by a transversal such that congruent corresponding angles are created, then the lines are parallel.

Given: $l \parallel m$, $m \parallel n$

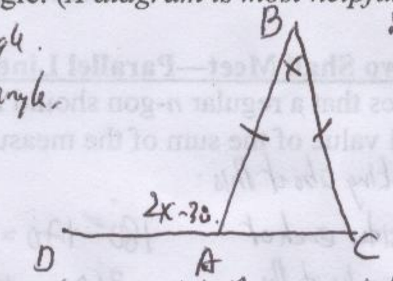
Prove: $l \parallel n$.

3) **(Normal/Hard)** In an isosceles triangle, an exterior angle drawn to one of the base angles measures 30 degrees less than double the measure of the vertex angle. Compute the numerical value of one of the base angles of this isosceles triangle. (*A diagram is most helpful here.*)

let x = measure of vertex angle.

$2x-30$ = measure of exterior angle.

* There are a few ways to do this. See if you can figure out others!



Note that an exterior angle of a triangle is equal to the sum of the two nonadjacent interior angles. Thus, $m\angle BAD = m\angle B + m\angle C$. So, we can therefore set $m\angle C = 2x-30-x = x-30$. Thus, $m\angle BAC = x-30$ as well. Since we have a linear pair, we get:

$m\angle BAD + m\angle BAC = 180$

$2x-30+x-30 = 180$

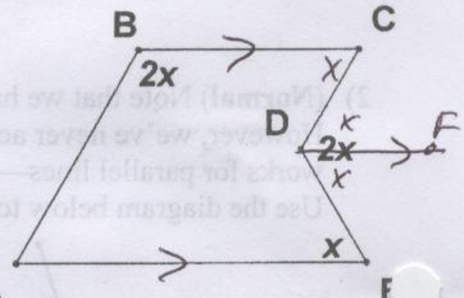
$3x-60 = 180$

$3x = 240$

$x = 80^\circ$

The vertex $\angle$ is 80°, so each base angle measures $\frac{1}{2}(180-80) = \frac{1}{2}(100) = \boxed{50^\circ}$.

4) **(Hard)** In the diagram shown at right, $\overline{AB} \parallel \overline{CD}$, $\overline{BC} \parallel \overline{AE}$, $m\angle ABC = m\angle CDE = 2x$, and $m\angle DEA = x$. Compute the value of x.



* Again, there are many ways to do this. Try to figure out others!

We construct a line $\overline{DF}$ parallel to $\overline{BC}$ and $\overline{AE}$. $\angle FDE$ and $\angle AED$ are alternate interior angles, so $m\angle FDE = x$. Thus, $m\angle CDF = 2x - x = x$. Since $\angle BCD$ and $\angle CDF$ are alternate interior angles, $m\angle BCD = x$ as well. Since $\overline{AB} \parallel \overline{CD}$, $\angle ABC$ and $\angle BCD$ are same-side interior angles, so they must be supplementary. Thus:

$2x + x = 180$

$3x = 180$

$\boxed{x = 60.}$

5) **(Phantasm)** For how many integer values of n will a regular n-gon have interior angles of integer measure? (*You could just count them all, but surely there's a better way…*)

Recall that the interior angle of a polygon is supplementary to its neighboring exterior angle. Thus, if the interior angle has an integer angle, so does the exterior angle. Since each exterior angle of an n-gon measures $\frac{360}{n}^\circ$, we simply need to find the values of n that make the fraction an integer. Thus, n needs to divide 360 evenly. There are 24 such integers that do so (how did we get that without listing? That's a story for another day…), and such it should be 24. However! If $n=1$ or $n=2$, that would not yield a polygon, since a polygon must have at least 3 sides. Thus, we discard two values of n, yielding

$24-2 = \boxed{22}$ values of n.