The Bronx High School of Science
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Course: MGS21(H) – (Honors) Geometry

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A Puff of Logic—Logic Review

1) (**Normal**) Are the statements $\sim p \wedge (p \to q)$ and $\sim (q \to p)$ logically equivalent? Justify your response.

p	q	$q \to p$	$\sim(q \to p)$	$\sim p$	$p \to q$	$\sim p \wedge (p \to q)$
T	T	T	F	F	T	F
T	F	T	F	F	F	F
F	T	F	T	T	T	T
F	F	T	F	T	T	T

no match!

Note that the two columns do not match. Therefore, the two statements are not logically equivalent.

2) (**Normal**) You are given that "*If I receive my bonus check, then we will go on a trip*", "*If my car breaks down, then we will not go on a trip*", "*Either I receive my bonus check or we will not buy souvenirs*", and "*My car breaks down.*" Prove that we will not buy souvenirs.

Let B = "I receive my bonus check."
T = "We go on a trip."
C: "My car breaks down."
S: "We will buy souvenirs."

Given: $B \to T$
$C \to \sim T$
$B \vee \sim S$
C
Prove: $\sim S$

Statements	Reasons
1. $C \to \sim T$	1. Given.
2. C	2. Given.
3. $\sim T$	3. Law of Detachment (1, 2)
4. $B \to T$	4. Given.
5. $\sim B$	5. Law of Modus Tollens (3, 4)
6. $B \vee \sim S$	6. Given.
7. $\sim S$	7. Law of Disjunctive Inference (5, 6)

3) (**Hard**) Is the statement $[\sim q \to (p \to q)] \vee (\sim p \wedge q)$ a tautology? Justify your answer.

p	q	$p \to q$	$\sim q$	$\sim q \to (p \to q)$	$\sim p$	$\sim p \wedge q$	$[\sim q \to (p \to q)] \vee (\sim p \wedge q)$
T	T	T	F	T	F	F	T
T	F	F	T	F	F	F	F
F	T	T	F	T	T	T	T
F	F	T	T	T	T	F	T

↑ not all T's.

Since the final column does not contain all Trues, the statement is not always true for all values of p and q. Thus, the statement is not a tautology.

4) **(Lunatic)** (*Hint: One variable is the linchpin for this entire proof. Once you figure out what it is, try to get it free.*)

Given: $s \to r$

$(p \vee q) \to \sim r$

$\sim s \to (\sim q \to r)$

p

Prove: q

It would be great if we knew that r was true - if that was the case, then we could get s and q! Perhaps we could detach the disjunction away...

Statements	Reasons
1. p	1. Given.
2. $p \vee q$	2. Law of Disjunctive Addition (1)
3. $(p \vee q) \to \sim r$	3. Given.
4. $\sim r$	4. Law of Detachment (2, 3)
5. $s \to r$	5. Given.
6. $\sim s$	6. Law of Modus Tollens (4, 5)
7. $\sim s \to (\sim q \to r)$	7. ~~Law of Detachment~~ Given.
8. $\sim q \to r$	8. Law of Detachment (6, 7)
9. q	9. Law of Modus Tollens (4, 8)

5) **(Lunatic)** You are given that the statement $[(\sim r \wedge s) \leftrightarrow (t \vee q)] \to (p \vee \sim q)$ is a false statement. What is the truth value of the statement s? Justify your answer.

The only time a conditional is false is if its hypothesis is true and its conclusion is false. Thus, $[(\sim r \wedge s) \leftrightarrow (t \vee q)]$ is true and $p \vee \sim q$ is false. Since $p \vee \sim q$ is false, this implies that both parts of the statement is false, so p is a false statement and q is a true statement. As the biconditional is true, both halves of the statement are the same truth value. Since q is true, t has no bearing - $t \vee q$ is always true. Thus, $\sim r \wedge s$ is true as well. As a conjunction is always true when both of its components are true,

s must be true.