

Law of Inference	Symbolic Forms	
1. Law of Detachment (Modus Ponens)	$\begin{array}{l} p \rightarrow q \\ p \\ \hline \therefore q \end{array}$	$[(p \rightarrow q) \wedge p] \rightarrow q$
2. Law of the Contrapositive	$\begin{array}{l} p \rightarrow q \\ \hline \therefore \sim q \rightarrow \sim p \end{array}$	$(p \rightarrow q) \leftrightarrow (\sim q \rightarrow \sim p)$
3. Law of Modus Tollens	$\begin{array}{l} p \rightarrow q \\ \sim q \\ \hline \therefore \sim p \end{array}$	$[(p \rightarrow q) \wedge \sim q] \rightarrow \sim p$
4. Chain Rule (Law of the Syllogism)	$\begin{array}{l} p \rightarrow q \\ q \rightarrow r \\ \hline \therefore p \rightarrow r \end{array}$	$[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$
5. Law of Disjunctive Inference	$\begin{array}{l} p \vee q \quad p \vee q \\ \sim p \quad \quad \sim q \\ \hline \therefore q \quad \therefore p \end{array}$	$\begin{array}{l} [(p \vee q) \wedge \sim p] \rightarrow q \\ [(p \vee q) \wedge \sim q] \rightarrow p \end{array}$
6. Law of the Double Negation	$\begin{array}{l} \sim(\sim p) \\ \hline \therefore p \end{array}$	$\sim(\sim p) \leftrightarrow p$
7. De Morgan's Laws	$\begin{array}{l} \sim(p \wedge q) \quad \sim(p \vee q) \\ \hline \therefore \sim p \vee \sim q \quad \therefore \sim p \wedge \sim q \end{array}$	$\begin{array}{l} \sim(p \wedge q) \leftrightarrow (\sim p \vee \sim q) \\ \sim(p \vee q) \leftrightarrow (\sim p \wedge \sim q) \end{array}$
8. Law of Simplification	$\begin{array}{l} p \wedge q \quad p \wedge q \\ \hline \therefore p \quad \therefore q \end{array}$	$\begin{array}{l} (p \wedge q) \rightarrow p \\ (p \wedge q) \rightarrow q \end{array}$
9. Law of Conjunction	$\begin{array}{l} p \\ q \\ \hline \therefore p \wedge q \end{array}$	$(p) \wedge (q) \rightarrow (p \wedge q)$
10. Law of Disjunctive Addition	$\begin{array}{l} p \\ \hline \therefore p \vee q \end{array}$	$p \rightarrow (p \vee q)$

EXERCISES

In 1–4, assign truth values to the variables to make the premises true. Then, demonstrate that the arguments are valid by showing that true premises must lead to a true conclusion.