The Bronx High School of Science
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Course: M3 – Regents Geometry

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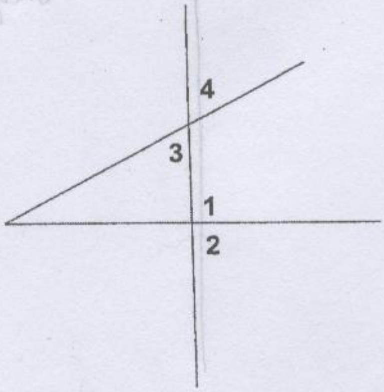
Proofs with Inequalities, Part II

The best way to work out a proof for inequalities is to think of the proof as a ladder to be completed—link the chains in order from a given to the prove statement. Note that the Prove statement is the last rung, and the givens can appear anywhere else in the ladder, but it's good to start off with at least one. If you're stuck in one direction, move in the other direction!

1)

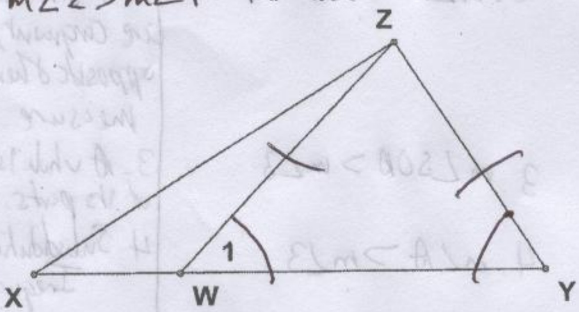


Statements	Reasons
1. $m\angle 2 > m\angle 1$	1. Given.
2. $m\angle 1 > m\angle 3$	2. An exterior angle of a triangle is greater than either of the remot[e] interior angles.
3. ~~$m\angle 3 = m\angle 4$~~ $\angle 3$ and $\angle 4$ are vertical angles.	3. Definition of vertical angles.
4. $m\angle 3 = m\angle 4$	4. Vertical angles have equal meas[ure]
5. $m\angle 1 > m\angle 4$	5. Substitution Postulate of Inequalities.
6. $m\angle 2 > m\angle 4$	6. Transitive Property of Inequality.

Given: $m\angle 2 > m\angle 1$
Prove: $m\angle 2 > m\angle 4$
Plan your chain here:

- $m\angle 2 > m\angle 1$ Given
- $m\angle 1 > m\angle 3$ – Ext. angle. Thm.
- $m\angle 3 = m\angle 4$ – vertical angles.
- $m\angle 1 > m\angle 4$ – Substitution
- $m\angle 2 > m\angle 4$ – Transitive. Prove.

2)

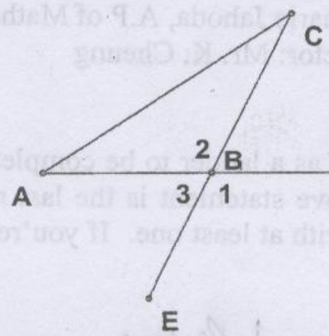


Statements	Reasons
1. $ZW = ZY$	1. Given.
2. $m\angle 1 = m\angle Y$	2. If two sides of a triangle are congruent, then the angles opposite them are equal in meas[ure]
3. $m\angle 1 > m\angle X$	3. The exterior angle of a trian[gle] is larger than either of the remot[e] interior angles.
4. $m\angle Y > m\angle X$	4. Substitution Postulate of Inequalities.
5. $XZ > YZ$	5. If two sides of a triangle ar[e] unequal, then the longer side is opposite the larger angle.

Given: $ZW = ZY$
Prove: $XZ > YZ$
(*Hint: Why is that 1 there?*)

$ZW = ZY$ Given
$m\angle 1 = m\angle Y$ – Isos. Δ Thm.
$m\angle 1 > m\angle X$ – Ext. Angle Thm.
$m\angle Y > m\angle X$ – Substitution Post.
$XZ > YZ$ – Larger angles opp. larger sides. Prove

3)

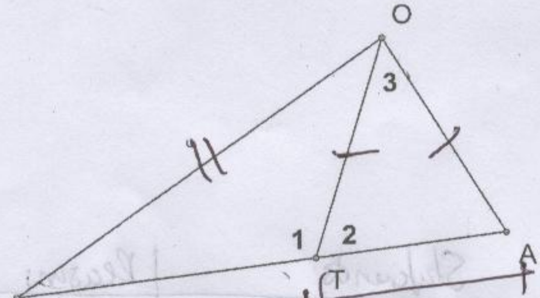


Statements	Reasons
1. $m\angle 2 > m\angle 3$	1. Given.
2. $m\angle 3 > m\angle A$	2. The measure of the exterior angle of a triangle is greater than either of the remote interior angles
3. $m\angle 2 > m\angle A$	3. Transitive Property of Inequality.

Given: $m\angle 2 > m\angle 3$
Prove: $m\angle 2 > m\angle A$

G. $m\angle 2 > m\angle 3$
$m\angle 3 > m\angle A$ - Ext angle Thm.
$m\angle 2 > m\angle A$ - Transitive Prop.

4)



Given: $SO = SA$
$OT = OA$
Prove: $OA > TA$

G: $SO = SA$
$m\angle A = m\angle SOA$ - Isos. $\triangle$ Thm.
$m\angle SOA > m\angle 3$ - Whole greater than parts.
$m\angle A > m\angle 3$ - Substitution Post.
$m\angle 2 > m\angle 3$ - Substitution Post.
P: $OA > TA$ - Larger sides opp larger angles.

Statements	Reasons
1. $SO = SA$	1. Given.
2. $m\angle A = m\angle SOA$	2. If two sides of a triangle are congruent, then the angles opposite them are equal in measure.
3. $m\angle SOA > m\angle 3$	3. A whole is greater than any of its parts.
4. $m\angle A > m\angle 3$	4. Substitution Postulate of Inequalities
5. $OT = OA$	5. Given
6. $m\angle 2 = m\angle A$	6. If two sides of a triangle are congruent, then the angles opposite them are equal in measure.
7. $m\angle 2 > m\angle 3$	7. Substitution Postulate of Inequalities.
8. $OA > TA$	8. If two sides of a triangle are unequal in measure, then the larger side is opposite the larger angle.

HW #52:
Blue Book p. 197-198 #18, 22
p. 203 #15, 16

~~$m\angle 1 > m\angle A$~~
$OT = OA$ - Given
$m\angle 2 = m\angle A$ - Isos $\triangle$ Thm.