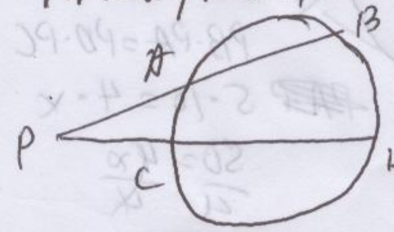
HW#25 Solutions:

p. 364-365 #3-5, 7, 9, 17-20, 27

For these problems, remember the version of Power of a Point that applies outside of a circle:

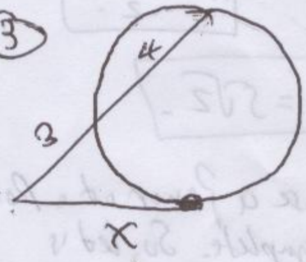


$PA \cdot PB = PC \cdot PD$

outside segments ; entire secants

This holds even if A and B are the same point.

(3)

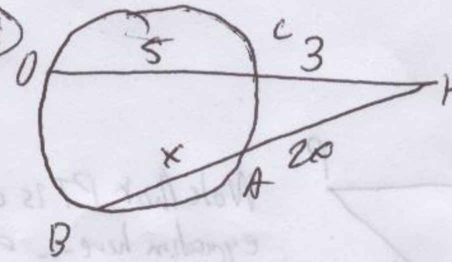


$x^2 = 3 \cdot 7$

$x^2 = 21$

$\boxed{x = \sqrt{21}.}$

(4)



$PA \cdot PB = PC \cdot PD$

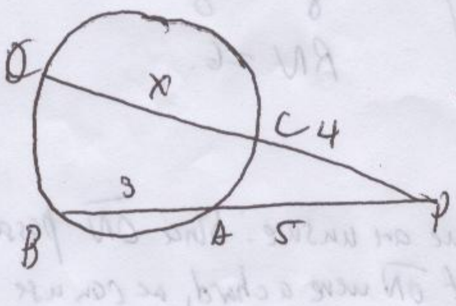
$2x(3x) = 3 \cdot 8$

$\frac{6x^2}{6} = \frac{24}{6}$

$x^2 = 4$

$\boxed{x = 2.}$

(5)



$PA \cdot PB = PC \cdot PD$

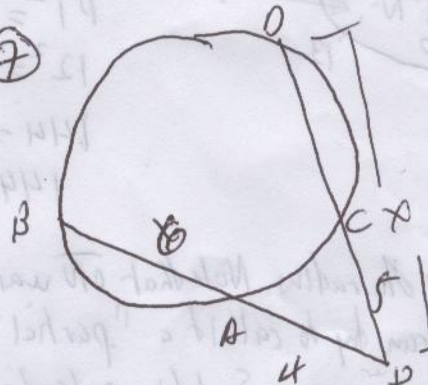
$5 \cdot 8 = 4(x+4)$

$40 = 4x + 16$

$\frac{24}{4} = \frac{4x}{4}$

$\boxed{6 = x.}$

(7)



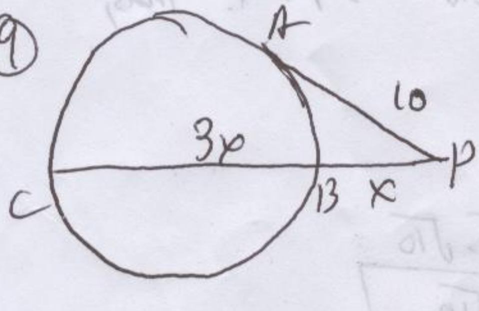
$PA \cdot PB = PC \cdot PD$

$4 \cdot 10 = 5 \cdot x$

$40 = 5x$

$\boxed{x = 8.}$

(9)



$PA^2 = PB \cdot PC$

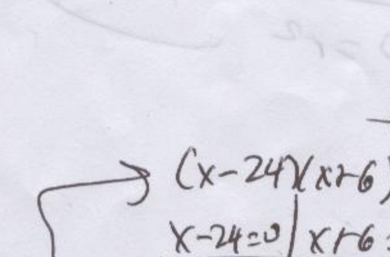
$10^2 = x \cdot 4x$

$\frac{100}{4} = \frac{4x^2}{4}$

$25 = x^2$

$\boxed{5 = x}$

(17)



$PT^2 = PB \cdot PA$

$6^2 = 3(3+x)$

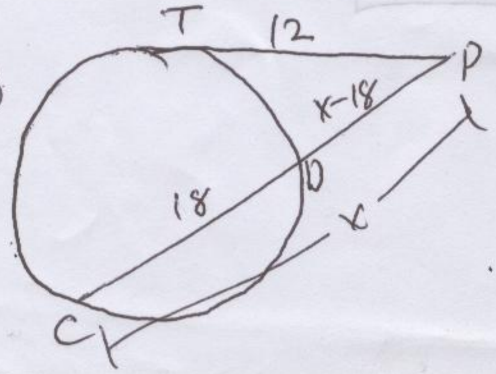
$6^2 = 9 + 3x$

$36 = 9 + 3x$

$\frac{27}{3} = \frac{3x}{3}$

$\boxed{9 = x.}$

(18)



$PT^2 = PD \cdot PC$

$12^2 = (x-18) \cdot x$

$144 = x^2 - 18x$

$x^2 - 18x - 144 = 0$

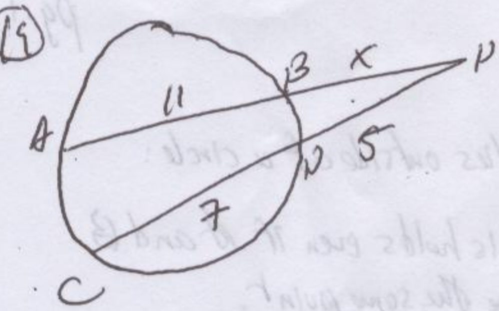
$(x-24)(x+6) = 0$

$x - 24 = 0 \quad | \quad x + 6 = 0$

$\boxed{x = 24} \quad | \quad x = -6$

reject, since lengths are not negative.

(19)



$PB \cdot PA = PD \cdot PC$

$x(x+11) = 5 \cdot 12$

$x^2+11x = 60$

$x^2+11x-60=0$

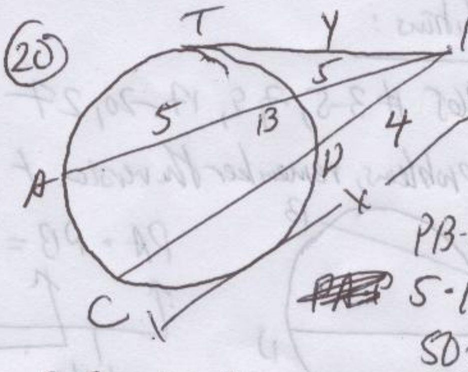
$(x+15)(x-4)=0$

$x+15=0 \quad | \quad x-4=0$

$x=-15 \quad | \quad \boxed{x=4}$

reject

(20)



$PB \cdot PA = PD \cdot PC$

$5 \cdot 10 = 4 \cdot x$

$\frac{50}{4} = \frac{4x}{4}$

$\boxed{x = \frac{25}{2}.}$

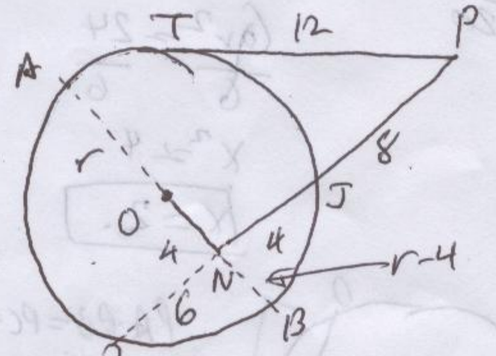
$PT^2 = PB \cdot PA$

$y^2 = 5 \cdot 10$

$y^2 = 50$

$\boxed{y = \sqrt{80} = \sqrt{25} \cdot \sqrt{2} = 5\sqrt{2}.}$

(27)



Note that PT is a tangent, so we'd love to use a Power of a Point equation here... except the secant is incomplete. So, let's complete it. Extend $\overline{PN}$ until it hits the circle again, at R. Thus,

$PT^2 = PJ \cdot PR$

$12^2 = 8 \cdot (8+4+RN)$

$144 = 8(12+RN)$

$144 = 96+8RN$

$8RN = 48$

$\frac{8RN}{8} = \frac{48}{8}$

$RN = 6.$

Now, to compute the radius. Note that $\overline{ON}$ was given, but for what, we are unsure. Since $\overline{ON}$ passes through the center, we can try to call it a "partial" radius. In fact, if $\overline{ON}$ were a chord, we can use Power of a Point again. So let's extend = extend $\overline{ON}$ until it hits the circle twice, at A and B. Let OA and OB, radii, be r. Thus, OA = r, and $NB = OB - ON \longrightarrow r-4$. Thus,

$RN \cdot NJ = AN \cdot NB$

$6 \cdot 4 = (r+4)(r-4)$

$24 = r^2 - 16$

$+16 \qquad +16$

$40 = r^2$

$\sqrt{40} = \sqrt{r^2}$

$r = \sqrt{40} = \sqrt{4} \cdot \sqrt{10}$

$\boxed{= 2\sqrt{10}.}$