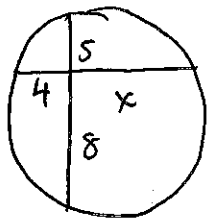
HW #23 Solutions:

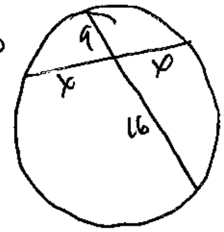
Orange Book p. 364–365 #1, 2, 6, 8, 13–16, 23, 24

① 

$4 \cdot x = 5 \cdot 8$

$\frac{\cancel{4}x}{\cancel{4}} = \frac{40}{4}$

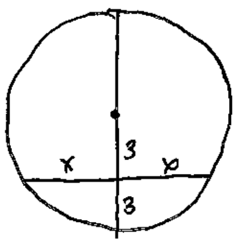
$\boxed{x = 10.}$

② 

$9 \cdot 16 = x \cdot x$

$144 = x^2$

$\boxed{x = 12.}$

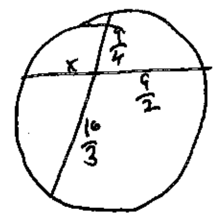
⑥ 

Note that the radius of the circle is 6, so the vertical section of the chord is 9. Thus:

$x \cdot x = 9 \cdot 3$

$x^2 = 27$

$\boxed{x = \sqrt{27} = 3\sqrt{3}.}$

⑧ 

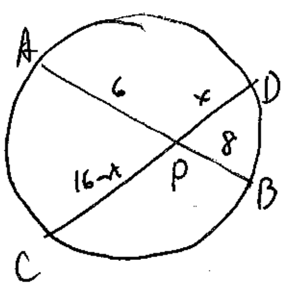
$x \cdot \frac{9}{2} = \frac{9}{4} \cdot \frac{16}{3}$

$\frac{9x}{2} = \frac{\overset{3}{\cancel{9}} \cdot \overset{4}{\cancel{16}}}{\cancel{4} \cdot \cancel{3}}$

$\frac{9x}{2} = 12$

$9x = 24$

$\boxed{x = \frac{24}{9} = \frac{8}{3}.}$

⑬ 

let $DP = x$,
$CP = 16 - x$.

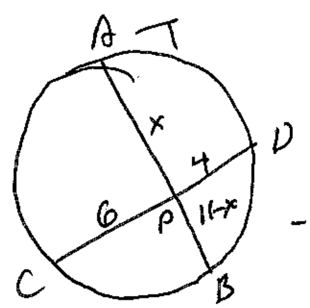
So: $6 \cdot 8 = x(16 - x)$

$48 = 16x - x^2$

$x^2 - 16x + 48 = 0$

$(x-4)(x-12) = 0$

$\boxed{x = 4 \text{ or } 12.}$

⑭ 

let $AP = x$, $PB = 11 - x$

$x(11 - x) = 6 \cdot 4$

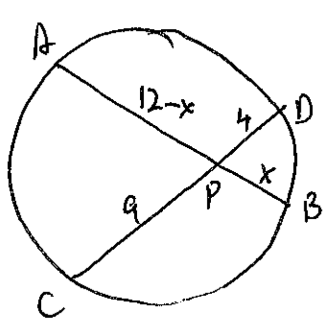
$-1(11x - x^2) = (24) - 1$

$x^2 - 11x = -24$

$x^2 - 11x + 24 = 0$

$(x-3)(x-8) = 0$

$\boxed{x = 3 \text{ or } 8.}$

⑮ 

let $PB = x$, $AP = 12 - x$

$(12 - x) \cdot x = 9 \cdot 4$

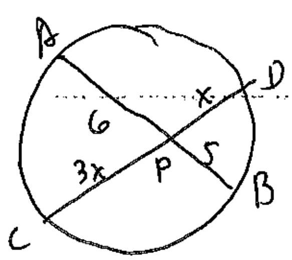
$12x - x^2 = 36$

$-x^2 + 12x - 36 = 0$

$x^2 - 12x + 36 = 0$

$(x-6)(x-6) = 0$

$\boxed{x = 6.}$

⑯ 

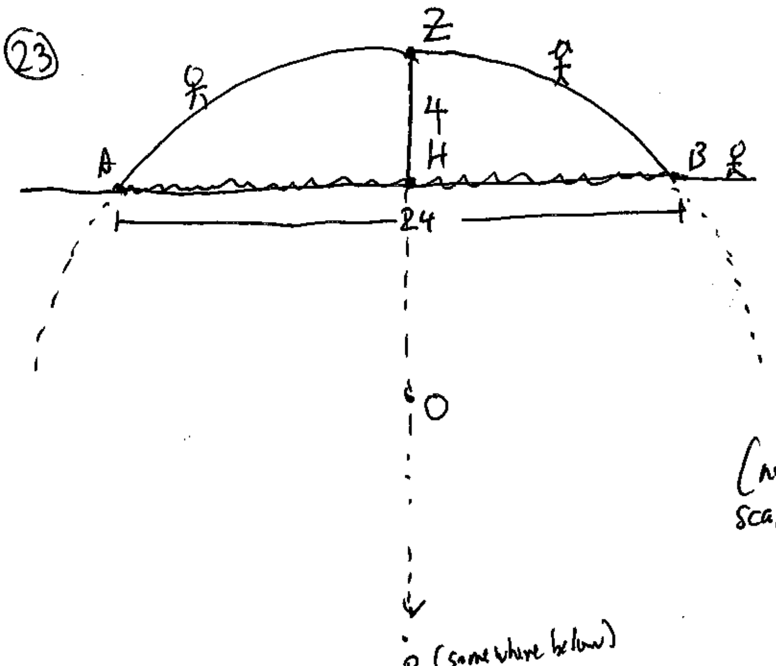
$6 \cdot 5 = x \cdot 3x$

$30 = 3x^2$

$10 = x^2$

$\boxed{x = \sqrt{10}.}$

(23)



(not to scale)

The center of the bridge's span (Z) ~~is the highest point of the bridge, and~~ as such should be directly over the center of the circle. If we were to continue the diameter and circle, we get diameter $\overline{ZB}$. Now, we can use the Power of a Point around H: Note that as $\overline{ZB}$ is a diameter, and it is a vertical line intersecting the horizontal span of the bridge $\overline{AB}$, it must bisect $\overline{AB}$, so $AH = HB = \frac{1}{2} \cdot 24 = 12$. We compute:

$AH \cdot HB = ZH \cdot HB$ ← $\overline{HB}$ is a diameter except for $\overline{ZH}$, which is 4.

$12 \cdot 12 = 4 \cdot (r + r - 4)$

$144 = 4(2r - 4)$

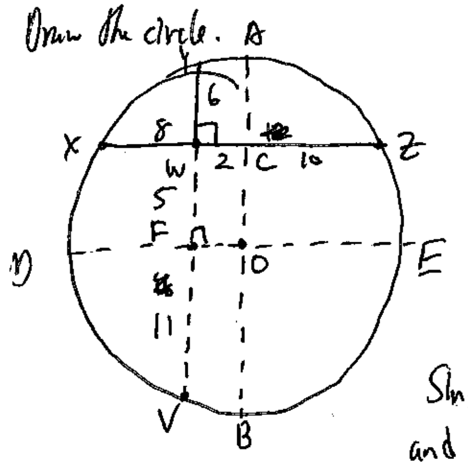
$144 = 8r - 16$

$160 = 8r$

$\boxed{r = 20.}$

The radius is 20 meters.

(24) Draw the circle.

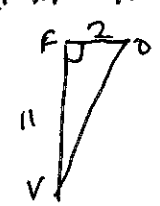


We would love to use Power of a Point on W, but the chord $\overline{YW}$ is incomplete. Let's complete it by extending $\overline{YW}$ to the circle at V. Now, by Power of a Point,

$XW \cdot WZ = YW \cdot WV$

$8 \cdot 12 = 6 \cdot WV \longrightarrow 96 = 6 \cdot WV \longrightarrow WV = 16.$

Since the given chords are perpendicular, without loss of generality, let $\overline{XZ}$ be horizontal and $\overline{YV}$ be vertical. Construct a diameter $\overline{AB}$ also perpendicular to $\overline{XZ}$ at a point C. Since $\overline{AB} \perp \overline{XZ}$, $\overline{AB}$ bisects $\overline{XZ}$, so $CZ = \frac{1}{2}(8+12) = 10$, so $WC = 10 - 8 = 2$. Construct a new diameter $\overline{DE}$ perpendicular to $\overline{YV}$ through F. By the same reasoning as above, $\overline{DE}$ bisects $\overline{YV}$, so $FV = \frac{1}{2}(6+16) = 11$, and $WF = 16 - 11 = 5$. Now consider $\triangle OFV$. It is right, so:



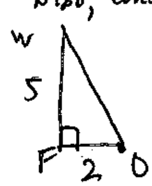
$OV^2 = r^2 = 2^2 + 11^2$

$r^2 = 4 + 121$

$r^2 = 125$

$r = \sqrt{125} = \boxed{5\sqrt{5}.}$

Also, consider right $\triangle WFO$:



$WO^2 = 5^2 + 2^2$

$WO^2 = 25 + 4$

$WO^2 = 29$

$\boxed{WO = \sqrt{29}.}$