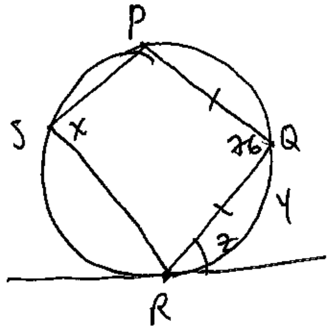
HW #21 Solutions:

GB: p. 354 # 7–9, 14, p. 359 written # 11–14.

p. 354:

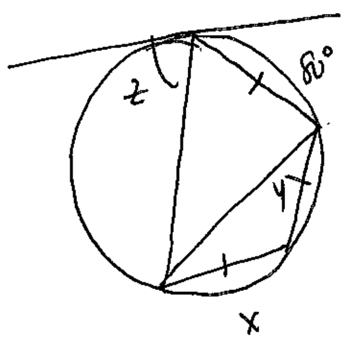
⑦



Note that in a previous solution set we proved that the opposite angles of any quadrilateral inscribed in a circle are supplementary. Thus, $x = 180 - 76 = 104°$. Now, note that x is an inscribed angle, so it measures half the arc that it intercepts. Thus, $m\,\widehat{PQR} = 2 \cdot 104 = 208°$. However, as $\overline{PQ} \cong \overline{QR}$, the arcs they intercept are congruent, so $\widehat{PQ} \cong \widehat{QR}$, and $m\,\widehat{QR} = m\,\widehat{PQ} = \frac{1}{2} \cdot 208 = 104°$. ~~Note~~ Thus $\underline{y = 104°}$. z is a chord-tangent angle that intercepts $\widehat{QR}$, so $z = \frac{1}{2} m\,\widehat{QR} = \frac{1}{2} \cdot 104° = \underline{\underline{52°}}$.

$$\boxed{\begin{aligned} x &= 104 \\ y &= 104 \\ z &= 52. \end{aligned}}$$

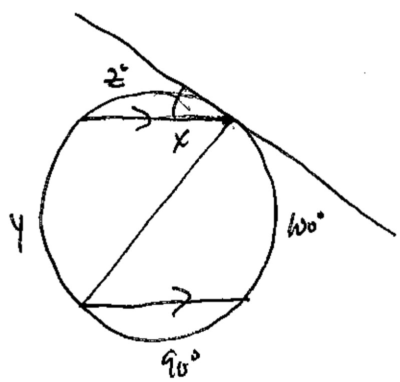
⑧



Note that the three marked chords are congruent. Thus, the arcs they intercept are too, so $\underline{x = 80°}$. y is an inscribed angle that intercepts an arc with measure x, so $y = \frac{1}{2} x = \frac{1}{2} \cdot 80 = \underline{40°}$. z is a chord-tangent angle, so it measures one-half its intercepted arc. The arc is thus $360 - 3 \cdot 80 = 360 - 240 = 120°$, so $z = \frac{1}{2} \cdot 120 = \underline{\underline{60°}}$.

$$\boxed{\begin{aligned} x &= 80 \\ y &= 40 \\ z &= 60. \end{aligned}}$$

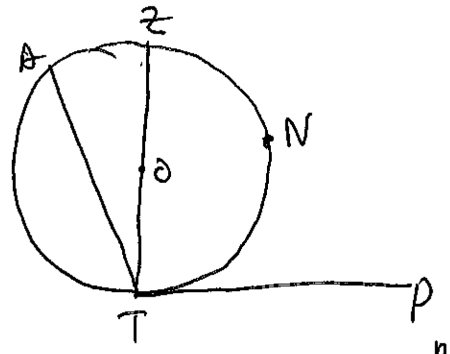
⑨



Note that the two given chords are parallel. Thus, the arcs that they cut off between them must be congruent, so $\underline{y = 100°}$. x is an inscribed angle, and thus measures $\frac{1}{2}$ of its intercepted arc, so $x = \frac{1}{2} \cdot 100 = \underline{50°}$. z, again a chord-tangent angle, is one-half of its intercepted arc. So, $z = \frac{1}{2}(360 - 100 - 90 - 100)$
$= \frac{1}{2}(70) = \underline{\underline{35°}}$.

$$\boxed{\begin{aligned} x &= 50 \\ y &= 100 \\ z &= 35. \end{aligned}}$$

(14)



Given: O lies inside $\angle ATP$.
Prove: $m\angle ATP = \frac{1}{2} m\overset{\frown}{ANT}$.

We draw diameter $\overline{TZ}$ through center O. Note now that $\angle ZTP$ is a chord-tangent angle, and by Case I established in problem #13, $m\angle ZTP = \frac{1}{2} m\overset{\frown}{ZNT}$. Since $\overline{ZT}$ is a diameter, $m\overset{\frown}{ZNT} = 180°$, so $m\angle ZTP = 90°$. Now, let $m\angle ATZ = x$. As $\angle ATZ$ is an inscribed angle, it measures half its intercepted arc, so $m\overset{\frown}{AZ} = 2x$.

Now, we will show:

$m\angle ATP = m\angle ATZ + m\angle ZTP$ by Partition.
$= x + 90.$

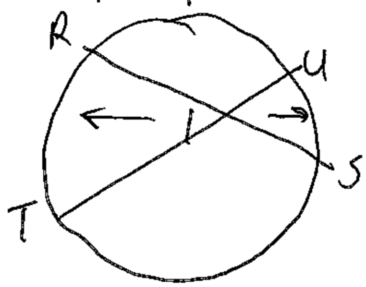
$m\overset{\frown}{ANT} = m\overset{\frown}{ZNT} + m\overset{\frown}{AZ}$ by Partition.
$= 180 + 2x.$

$m\angle ATP \overset{?}{=} \frac{1}{2} m\overset{\frown}{ANT}$
$x + 90 \overset{?}{=} \frac{1}{2}(180 + 2x) \longrightarrow x + 90 = 90 + x \longrightarrow x + 90 = x + 90$ ✓

QED.

p.359

Here, always remember for this set that



$$m\angle 1 = \frac{m\overset{\frown}{RT} + m\overset{\frown}{US}}{2}.$$

(11) $m\overset{\frown}{RT} = 80$, $m\overset{\frown}{US} = 40 \longrightarrow m\angle 1 = \frac{80+40}{2} = \frac{120}{2} = \boxed{60°.}$

(12) $m\overset{\frown}{RU} = 130$, $m\overset{\frown}{TS} = 100 \longrightarrow m\overset{\frown}{RT} + m\overset{\frown}{US} = 360 - (m\overset{\frown}{RU} + m\overset{\frown}{TS})$
$= 360 - (130 + 100)$
$= 360 - 230 = 130°.$

$m\angle 1 = \frac{130}{2} = \boxed{65°.}$

(13) $m\angle 1 = 50$, $m\overset{\frown}{RT} = 70 \longrightarrow 50 = \frac{70 + m\overset{\frown}{US}}{2} \longrightarrow 100 = 70 + m\overset{\frown}{US} \Longrightarrow m\overset{\frown}{US} = \boxed{30°.}$

(14) $m\angle 1 = 52$, $m\overset{\frown}{US} = 36 \longrightarrow 52 = \frac{m\overset{\frown}{RT} + 36}{2} \longrightarrow 104 = m\overset{\frown}{RT} + 36 \longrightarrow m\overset{\frown}{RT} = \boxed{68°.}$