HW #20 Solutions

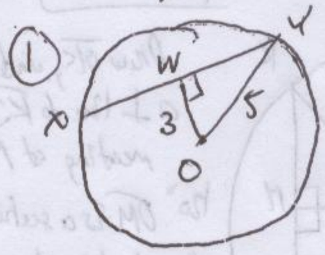
R: p. 347 #1–9, 11, 12, 17, 18

H: p. 347 #1–9, 12, 13, 17, 18, 20

The key here is to remember two big things:

* A radius (or diameter) perpendicular to a chord bisects that chord and its angles.
* Congruent chords cut off congruent arcs in the same circle. The reverse is true too!

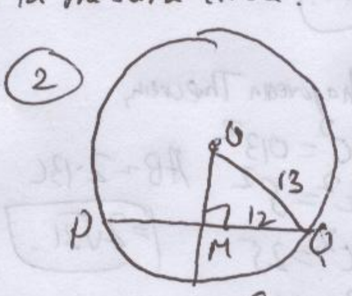
(1)



WY = 4, so $\boxed{XY = 8.}$

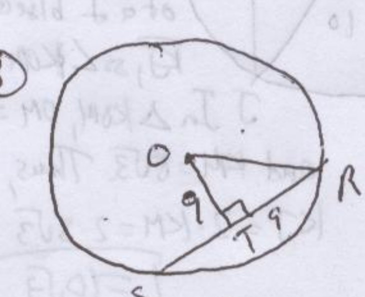
(use 3-4-5 rt $\triangle$ here.)

(2)



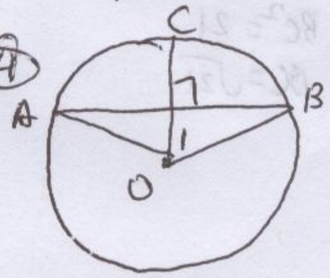
PQ = 24, so PM = 12.
Thus, $\boxed{OM = 5}$ using the 5-12-13 right $\triangle$.

(3)



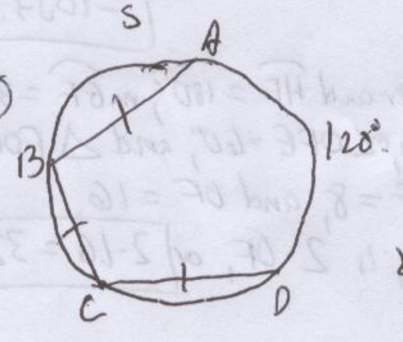
RS = 18, so TR = 9.
Thus, $\triangle ORT$ is isosceles right, so OR = $\boxed{9\sqrt{2}.}$

(4)



$m\widehat{ACB} = 110$. Thus, central $\angle AOB = 110°$.
Thus, $\boxed{m\angle 1 = \frac{1}{2} \cdot 110° = 55°.}$

(5)



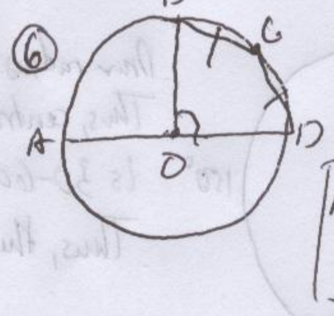
Here, $\overline{AB} \cong \overline{BC} \cong \overline{CD}$.
Thus, $\widehat{AB} \cong \widehat{BC} \cong \widehat{CD}$.
Let $m\widehat{BC} = x$. So,

$x + x + x + 120 = 360$

$\frac{3x}{3} = \frac{240}{3}$

$\boxed{x = 80°.}$

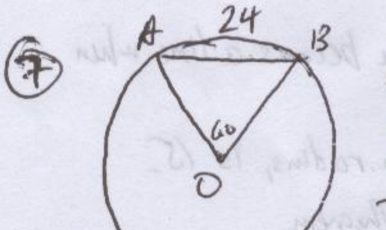
(6)



$m\angle BOD = m\widehat{BD} = 90°$.
Since $\overline{BC} \cong \overline{CD}$,
$\widehat{BC} \cong \widehat{CD}$. Thus,

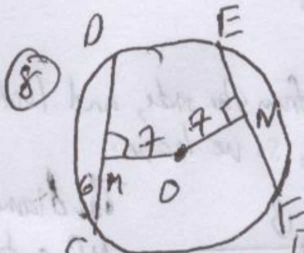
$\boxed{m\widehat{CD} = \frac{1}{2} m\widehat{BD} = \frac{1}{2} \cdot 90 = 45°.}$

(7)



$\overline{AO} \cong \overline{OB}$ since they are both radii. Thus, as the vertex angle is 60°, $\triangle AOB$ is equilateral.
Thus, $\boxed{OA = AB = 24.}$

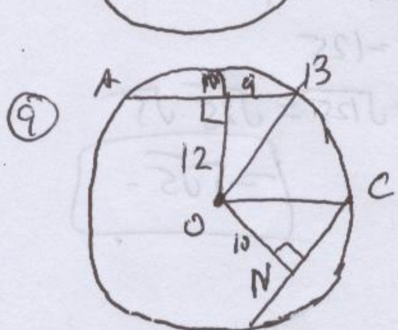
(8)



Here, $\overline{OM} \cong \overline{ON}$. Thus, $\overline{CD}$ and $\overline{EF}$ are equidistant from O, so $\overline{CD} \cong \overline{EF}$.
As DC = 2·MC, and DC = EF,

$\boxed{EF = 2 \cdot MC = 2 \cdot 6 = 12.}$

(9)



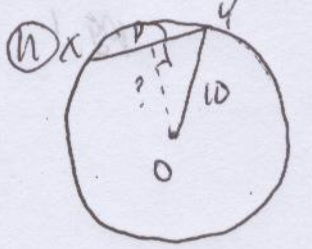
Here, this is a bit involved. Draw $\overline{OB}$ and $\overline{OC}$. $\triangle OMB$ is right. $MB = \frac{1}{2} AB =$
Thus, OB = 15, as it is triple a 3-4-5 right $\triangle$. So, OC = 15 since it is also a radius. By Pythagorean Theorem,

$NC^2 + ON^2 = OC^2$

$NC^2 + 10^2 = 15^2$

$NC^2 + 100 = 225 \rightarrow NC^2 = 125 \rightarrow NC = \sqrt{125} = \sqrt{25} \cdot \sqrt{5} = 5\sqrt{5}.$

Thus, $CD = 2 \cdot NC = 2 \cdot 5\sqrt{5}$ $\boxed{= 10\sqrt{5}.}$

(11)


$XY = 8$, so $PY = 4$. Thus, by Pythagorean Theorem,

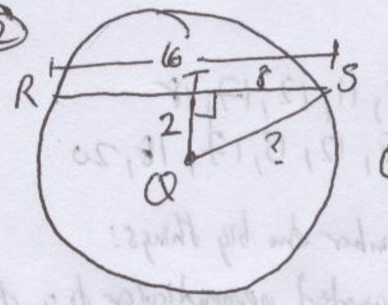
$PY^2 + PO^2 = OY^2$

$4^2 + PO^2 = 10^2$

$16 + PO^2 = 100$

$\sqrt{PO^2} = \sqrt{84} = \sqrt{4} \cdot \sqrt{21}$

$= \boxed{2\sqrt{21}.}$

(12)


$TS = 8$, as it is ½ o[f] RS. So,

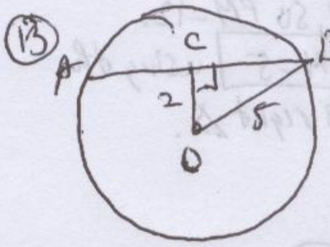
$QS^2 = TS^2 + QT^2$

$= 8^2 + 2^2$

$= 64 + 4 = 68$

$QS = \sqrt{68} = \sqrt{4} \cdot \sqrt{17}$

$\boxed{= 2\sqrt{17}}$

(13)


By Pythagorean Theorem,

$OC^2 + BC^2 = OB^2$

$2^2 + BC^2 = 5^2$

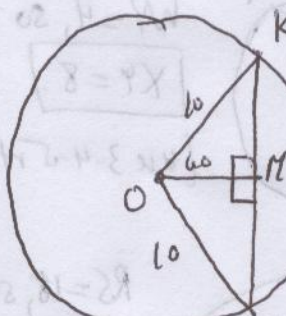
$4 + BC^2 = 25$

$BC^2 = 21$

$BC = \sqrt{21}.$

$AB = 2 \cdot BC$

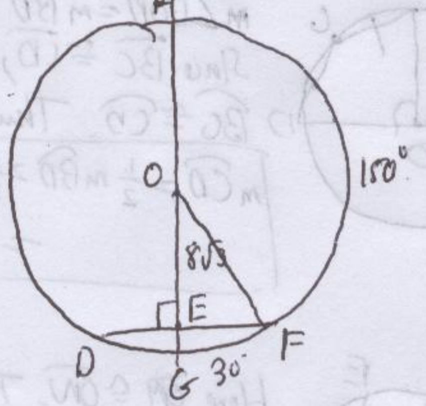
$\boxed{= 2\sqrt{21}.}$

(17)


Draw $\overline{OK}$, and a ⊥ line to $\overline{KJ}$ meeting at M. 120° OM is a section of a ⊥ bisector o[f] $\overline{KJ}$, so $\angle KOM = 60$. In $\triangle KOM$, $OM = 5$, and $KM = 5\sqrt{3}$. Thus,

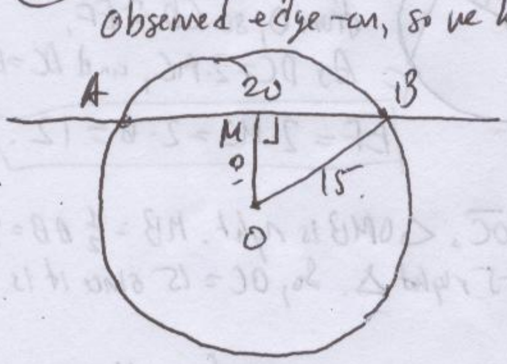
$KJ = 2 \cdot KM = 2 \cdot 5\sqrt{3}$

$\boxed{= 10\sqrt{3}.}$

(18)


Draw radius $\overline{OF}$. Since $\overline{HG}$ is a diameter and $\widehat{HF} = 180°$, $m\widehat{GF} = 30°$. Thus, central $\angle GOF = 30°$ as well. So, $m\angle OFE = 60°$, and $\triangle FOE$ is 30-60-90. By observation, $EF = 8$, and $OF = 16$. Thus, the length of $\overline{HG}$, a diameter, is $2 \cdot OF$, or $\boxed{2 \cdot 16 = 32.}$

(20) Slice the sphere from the side, and look at it from the side. The plane becomes a line when observed edge-on, so we have:



The diameter of the sphere is 30, so $\overline{OB}$, the radius, is 15. $MB = \frac{1}{2} AB = 10$. By Pythagorean Theorem,

$MO^2 + MB^2 = OB^2$

$MO^2 + 10^2 = 15^2$

$MO^2 + 100 = 225 \rightarrow MO^2 = 125$

$MO = \sqrt{125} = \sqrt{25} \cdot \sqrt{5}$

$\boxed{= 5\sqrt{5}.}$