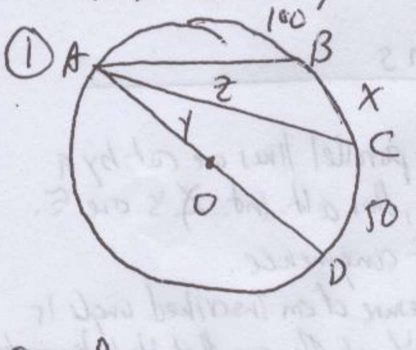
HW#19:

Orange Book p. 354 #1-4, 6, 10, 19

Remember two important facts here:

* An inscribed angle measures $\frac{1}{2}$ of its intercepted arc.

* A central angle is equal in measure to its intercepted arc.

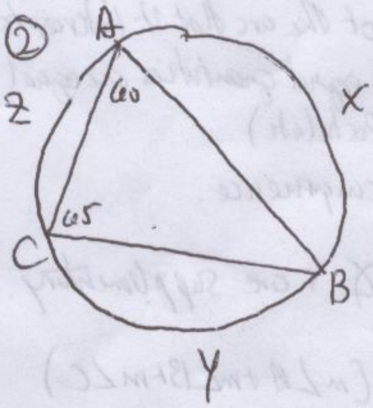
1. 

$\overline{AD}$ is a diameter, so $m\widehat{ABD} = 180°$. Since $m\widehat{AB} = 100$, $m\widehat{BD} = 80°$.

Since $m\widehat{BD} = x + 50$, $x + 50 = 80$, so $\boxed{x = 30}$.

$y = m\angle CAD = \frac{1}{2} m\widehat{CD} = \frac{1}{2} \cdot 50 = \boxed{25°}$

$z = m\angle BAC = \frac{1}{2} m\widehat{BC} = \frac{1}{2} \cdot 30 = \boxed{15°}$

2. 

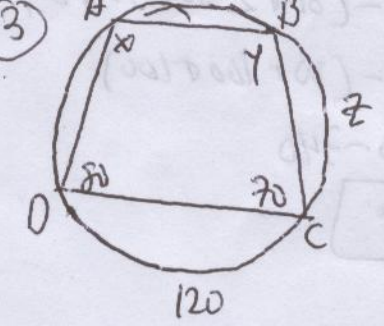
$X = m\widehat{AB}$.

$m\angle ACB = \frac{1}{2} m\widehat{AB} \longrightarrow 65 = \frac{1}{2} \cdot x \longrightarrow \boxed{x = 130°}$

$m\angle CAB = \frac{1}{2} m\widehat{BC} \longrightarrow 60 = \frac{1}{2} \cdot y \longrightarrow \boxed{y = 120°}$

$x + y + z = 360$, as this generates a full circle. So:

$z = 360 - x - y = 360 - 130 - 120 = 230 - 120 = \boxed{110°}$

3. 

Here, we use the fact that opposite angles of a quadrilateral inscribed in a circle is 180°. We prove it as such:

$m\angle A = x = \frac{1}{2} m\widehat{BCD}$. $m\angle C = 70 = \frac{1}{2} m\widehat{DAB}$.

$m\angle A + m\angle C = x + 70 = \frac{1}{2}(m\widehat{BCD} + m\widehat{DAB}) \longleftarrow$ full circle.

$= \frac{1}{2}(360) = 180.$

$x + 70 = 180 \longrightarrow \boxed{x = 110.}$

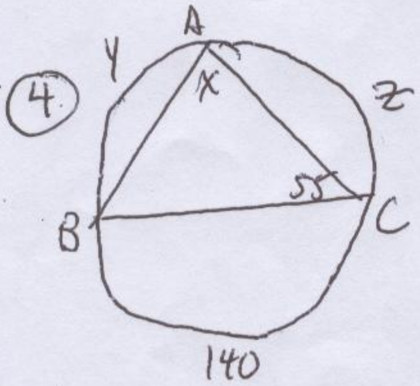
$y + 80 = 180$ by same reasoning. $\longrightarrow \boxed{y = 100.}$

$x = m\angle A = \frac{1}{2} m\widehat{BCD} = \frac{1}{2}(z + 120)$

$110 = \frac{1}{2}(z + 120)$

$110 = z/2 + 60 \to z/2 = 50$

$\boxed{z = 100.}$

4. 

$m\angle BAC = \frac{1}{2} m\widehat{BC} \longrightarrow x = \frac{1}{2} \cdot 140$

$\boxed{x = 70°.}$

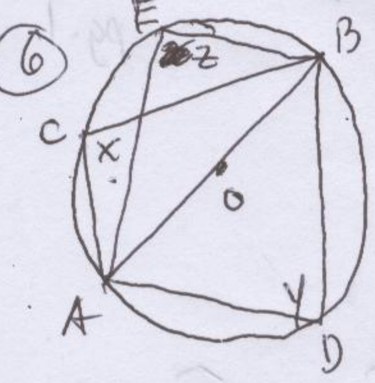
$m\angle C = 55 = \frac{1}{2} m\widehat{AB} = \frac{1}{2} \cdot y \longrightarrow \boxed{y = 110°.}$

$y + z + 140 = 360 \longrightarrow 110 + z + 140 = 360$

$250 + z = 360$

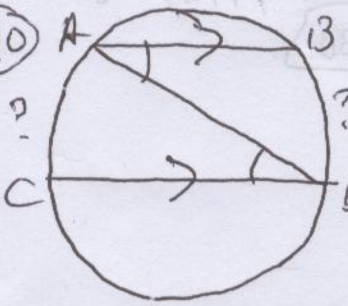
$\boxed{z = 110°.}$

(6)



Here, $\boxed{x = y = z = 90^\circ}$ By Thales' Theorem, each angle $\angle C$, $\angle D$, and $\angle E$ all intercept a semicircle, since $\overline{AB}$ is a diameter.

(10)

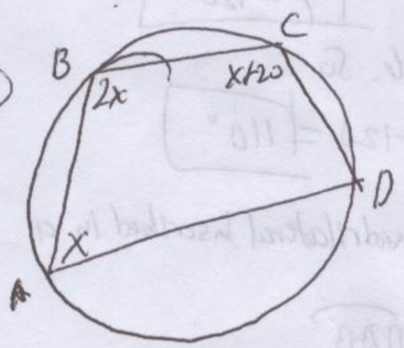


Given: $\overline{AB} \parallel \overline{CD}$

Prove: $\widehat{AC} \cong \widehat{BD}$

Statements	Reasons
1. $\overline{AB} \parallel \overline{CD}$	1. Given.
2. $\angle A \cong \angle D$	2. If two parallel lines are cut by a transversal, then alt. int. ∡'s are ≅.
3. $m\angle A = m\angle D$	3. Def. of congruence.
4. $m\angle A = \frac{1}{2} m\widehat{BD}$ $m\angle D = \frac{1}{2} m\widehat{AC}$	4. The measure of an inscribed angle is equal to ½ of the arc that it intercepts
5. $m\widehat{BD} = m\widehat{AC}$	5. Halves of equal quantities are equal (Division Postulate)
6. $\widehat{BD} \cong \widehat{AC}$	6. Def. of congruence.

(18)



Refer to #3 for an explanation of why opp. ∡'s are supplementary.

$m\angle A + m\angle C = 180^\circ$

$x + x + 20 = 180$

$2x + 20 = 180$

$-20 \quad -20$

$\frac{2x}{2} = \frac{160}{2}$

$\boxed{x = 80}$

$m\angle D = 360 - (m\angle A + m\angle B + m\angle C)$

$= 360 - (80 + 2\cdot 80 + (80 + 20))$

$= 360 - (80 + 160 + 100)$

$= 360 - 340$

$= \boxed{20^\circ}$