The Bronx High School of Science
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Course: M3(H) – (Honors) Geometry

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H.W #6

For problems #1-4, use:

Let *r* be "It rains."
Let *s* be "It snows."
Let *v* be "Vegetables grow."

Express the given statements in symbolic form, write the conclusion in symbolic form, and translate it into words.

1) If it rains, then vegetables grow. $r \to v$
   It rains. r
   By Law of Detachment, $\boxed{v\text{, or "Vegetables grow."}}$

2) If it snows, then vegetables do not grow. $s \to \sim v$
   It snows. s
   By Law of Detachment, $\boxed{\sim v\text{, or "Vegetables do not grow."}}$

3) If it does not rain and it does not snow, then vegetables grow. $(\sim r \wedge \sim s) \to v$
   Vegetables do not grow. $\sim v$
   By Modus Tollens, $\sim(\sim r \wedge \sim s) \equiv r \vee s$, or "It rains or it snows."

4) If it rains and it does not snow, then vegetables grow. $(r \wedge \sim s) \to v$
   It rains and it does not snow. $r \wedge \sim s$
   By Law of Detachment, v, or "vegetables grow."

For each of the following problems, write a valid conclusion based on the premises, or if no conclusion is possible, state so.

5) If I am smart, then I like chemistry.
   I do not like chemistry.
   $\therefore$ I am not smart by Modus Tollens.

6) If today is Friday, then tomorrow is not a school day.
   Today is Friday.
   $\therefore$ Tomorrow is not a school day by Detachment.

7) If I am tired, then I sleep.
   I am not sleeping.
   $\therefore$ I am not tired by Modus Tollens.

8) If I fill up this notebook, then I must buy another notebook.
   I must buy another notebook.
   This is an invalid argument, so no conclusion.

9) If Bob wakes up early, then he will walk to school.
   Bob did not wake up early.
   No conclusion, as it is invalid.

10) We talked about why the Law of Modus Tollens works with a truth value argument. Now, let's explain it with a truth table. Show that the Law of Modus Tollens is valid by constructing a truth table for

$$[(p \to q) \wedge \sim q] \to \sim p$$

and showing that this is a tautology.

Next.

(10)

p	q	$p \to q$	$\sim q$	$(p \to q) \wedge \sim q$	$\sim p$	$[(p \to q) \wedge \sim q] \to \sim p$
T	T	T	F	F	F	T
T	F	F	T	F	F	T
F	T	T	F	F	T	T
F	F	T	T	T	T	T

tautology.

As $[(p \to q) \wedge \sim q] \to \sim p$ is a tautology, the Law of Modus Tollens is valid. $\square$