The Bronx High School of Science
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Course: MGS21(H) – (Honors) Geometry

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*Remember: $\sim(p\wedge q) \equiv \sim p \vee \sim q$
$\sim(p\vee q) \equiv \sim p \wedge \sim q$

Logical Reasoning: Putting it All Together

Use DeMorgan's Laws to write a statement that is logically equivalent to the following:

1) It is not the case that Tommy is late for work and Jimmy is sick today.

Tommy is not late for work or Jimmy is not sick today.

2) $\sim(\sim s \vee \sim t)$ $\boxed{s \wedge t}$

3) $\sim(w \wedge \sim r)$ $\boxed{\sim w \vee r}$

4) What is the negation of the statement $(r \vee \sim t)\wedge(\sim p \wedge \sim q)$? Apply the law twice in succession:

$\sim[(r\vee\sim t)\wedge(\sim p\wedge\sim q)]$

$\sim(r\vee\sim t) \vee \sim(\sim p\wedge \sim q)$ → $\boxed{(\sim r\wedge t)\vee(p\wedge q)}$

5) Fill in the blank with the appropriate: $\sim p$ q

The statement "*Jim does not run for mayor and Jim runs for governor*" is logically equivalent to the statement

"It is not true that Jim runs for mayor or Jim does not run for governor. ."

We "undo" the DeMorgan's Law:

$\sim p \wedge q \equiv \sim(p\vee\sim q)$.

6) DeMorgan's Laws are great for conjunctions and disjunctions. Does a similar version exist for the conditional—that is, is the statement $\sim(r\rightarrow t)$ logically equivalent to the statement $\sim r \rightarrow \sim t$? What about $\sim t \rightarrow \sim r$? Justify your answers with a truth table.

We determine this via truth table:

r	t	$r\rightarrow t$	$\sim(r\rightarrow t)$	$\sim r$	$\sim t$	$\sim r\rightarrow\sim t$	$\sim t\rightarrow\sim r$
T	T	T	F	F	F	T	T
T	F	F	T	F	T	T	F
F	T	T	F	T	F	F	T
F	F	T	F	T	T	T	T

Note that none of the columns match exactly — thus none of the target statements are logically equivalent to the original. So:

$$\sim(r\rightarrow t) \not\equiv \sim r \rightarrow \sim t$$
$$\not\equiv \sim t \rightarrow \sim r$$

Not logically equivalent to.

7) When asked about whether Mr. Cheung gives pop quizzes, Mr. Cheung replies with the statement "*If I am in a very bad mood, then I will give you a pop quiz.*" The following day, the same class asks Mr. Cheung about the same subject, and he replies "*I am not in a very bad mood or I will give you a pop quiz.*" Do you think Mr. Cheung is telling you the same thing (albeit in different ways)? Justify your answer.

Let p: I am in a very bad mood and q: I will give you a pop quiz.
The target statements translate to $p \to q$ and $\sim p \vee q$. We test by truth table:

p	q	$p \to q$	$\sim p$	$\sim p \vee q$
T	T	T	F	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

match!

Since the target statements match in truth values, the two statements must be logically equivalent. Thus, Mr. Cheung is telling you the same thing.

Fill in the truth table below:

p	q	$\sim p$	$\sim p \vee q$	$p \wedge (\sim p \vee q)$	$p \wedge q$	$[p \wedge (\sim p \vee q)] \leftrightarrow (p \wedge q)$
T	T	F	T	T	T	T
T	F	F	F	F	F	T
F	T	T	T	F	F	T
F	F	T	T	F	F	T

Based on the table above, answer the following questions:

8) Which two statements in the table are logically equivalent? Justify your answer.

$\boxed{p \wedge q \text{ and } p \wedge (\sim p \vee q)}$ are logically equivalent since they always have the same truth values.

9) What does the last column tell you about the statement $[p \wedge (\sim p \vee q)] \leftrightarrow (p \wedge q)$? Justify your answer.

This statement is a tautology since it is always a true statement.

10) Based on what you observed in problems 8 and 9, complete the following statement.
"Aside from having their truth values being identical, two statements are logically equivalent if

a biconditional involving both statements yields a tautology."

For problems 11-16, assume that p is a false statement, q is a true statement, but you do not know the truth value of statement r. Determine the truth values of each of the following statements, if possible.

11) $p \leftrightarrow q$ F ↔ T

$\boxed{\text{False.}}$

12) $q \vee r$ T ∨ ?

$\boxed{\text{True}}$, since q is already true.

13) $p \to r$ F → ?

$\boxed{\text{True}}$ since no matter what r is, the statement is true:
F → F : True, F → T : True.

14) $q \to r$ T → ?

$\boxed{\text{Unknown}}$, since if $r = T$, the statement is true but if $r = F$, the statement is false.

15) $r \leftrightarrow r$

$\boxed{\text{True}}$, since despite r being unknown, it always has a matching truth value with itself.

16) $p \wedge (q \vee r)$

F ∧ (T ∨ ?)

$\boxed{\text{False.}}$

From #12, $q \vee r$ is true. Thus, we get F ∧ T, which is a false statement.