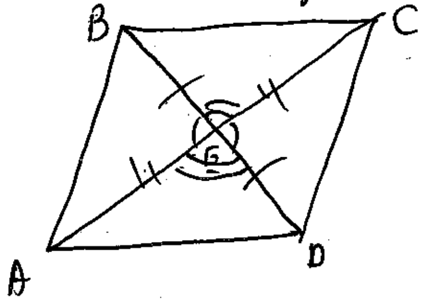
# HW #51 Solutions (Proofs Only)

OB p. 175 # 13, 14, 18

*Note: Your proofs may vary.

(13) Thm: If the diagonals of a parallelogram bisect each other, then the quadrilateral is a parallelogram.

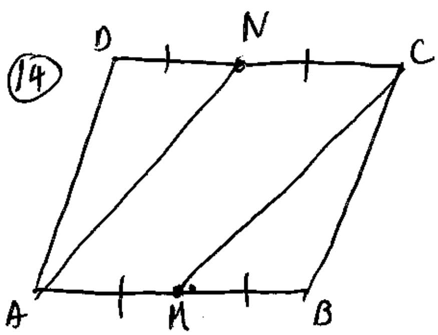



Given: $\overline{AC}$ bisects $\overline{BD}$
$\overline{BD}$ bisects $\overline{AC}$

Prove: ABCD is a parallelogram.

| Statements | Reasons |
|---|---|
| 1. $\overline{AC}$ bisects $\overline{BD}$, $\overline{BD}$ bisects $\overline{AC}$. | 1. Given. |
| 2. E is the midpoint of $\overline{AE}$ and $\overline{BD}$. | 2. Definition of segment bisector. (1) |
| 3. $\overline{BE} \cong \overline{ED}$, $\overline{AE} \cong \overline{EC}$ (s ≅ s) (s ≅ s) | 3. Definition of midpoint. |
| 4. $\angle BEC \cong \angle AED$, (a ≅ a) $\angle CED \cong \angle AEB$ (a ≅ c) | 4. Vertical angles are congruent. |
| 5. $\triangle BEC \cong \triangle DEB$, $\triangle BEA \cong \triangle DEC$ | 5. SAS Postulate (3, 4) |
| 6. $\overline{BC} \cong \overline{AD}$, $\overline{BA} \cong \overline{DC}$ | 6. Corresponding parts of congruent triangles are congruent. (5). |
| 7. ABCD is a parallelogram. | 7. If a quadrilateral has two pairs of opp. congruent sides, then it is a parallelogram. (6). |

(14)




Given: ABCD is a parallelogram
M and N are the midpoints of $\overline{AB}$ and $\overline{DC}$ respectively.

Prove: AMCN is a parallelogram.

We are given that ABCD is a parallelogram. Thus, $\overline{AB} \parallel \overline{CD}$ since opposite sides of a parallelogram are parallel. As $\overline{AM}$ and $\overline{NC}$ are subsegments of $\overline{AB}$ and $\overline{DC}$ respectively, $\overline{AM} \parallel \overline{NC}$. Now, we know that M and N are midpoints of $\overline{AB}$ and $\overline{DC}$ respectively. Thus, $NC = \frac{1}{2} DC$, and $AM = \frac{1}{2} AB$, since a midpoint divides a segment into two segments half the length of the original. We know that $\overline{AB} \cong \overline{DC}$, and thus $AB = DC$, since opposite sides of a parallelogram are congruent. Thus, by Division Postulate, $AM = NC$, so $\overline{AM} \cong \overline{NC}$. As AMCN has a pair of sides ($\overline{AM}$ and $\overline{NC}$) that is both congruent and parallel, AMCN is a parallelogram.

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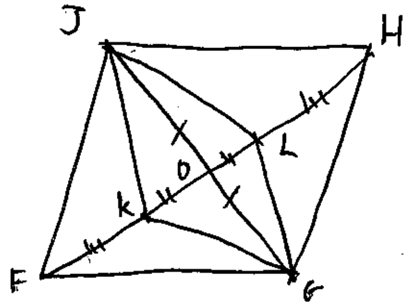


Given: KGLJ is a parallelogram

$FK = HL$

Prove: FGHJ is a parallelogram.

We know that KGLJ is a parallelogram. Thus, $\overline{JG}$ bisects and $\overline{KL}$ bisects $\overline{JG}$, as the diagonals of a parallelogram bisect each other. We focus our attention to $\overline{JG}$ bisecting $\overline{KL}$. By definition of a segment bisector, O is the midpoint of $\overline{KL}$. Thus, $\overline{OK} \cong \overline{OL}$ by definition of midpoint. Converting our other given, $FK = HL$, into a congruence statement, $\overline{FK} \cong \overline{HL}$. By Addition Postulate, $\overline{FK} + \overline{OK} \cong \overline{HL} + \overline{OL}$, or $\overline{FO} \cong \overline{HO}$. We reverse our definition chain to conclude that $\overline{JG}$ bisects $\overline{FH}$ as well. Thus, since FGHJ has diagonals that bisect each other, FGHJ is a parallelogram.