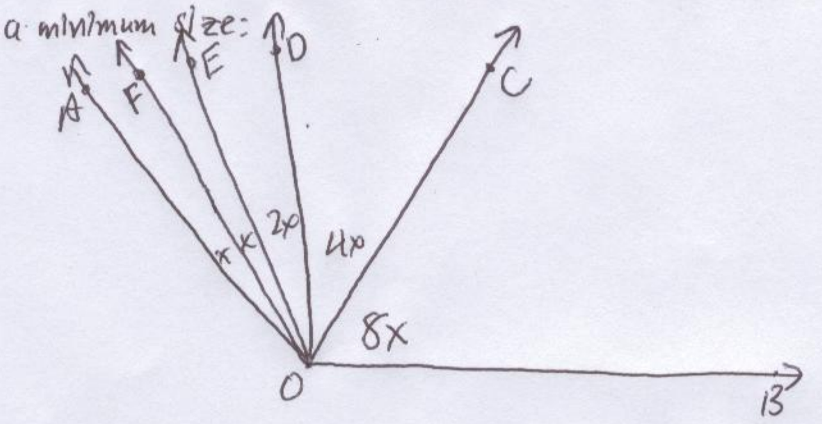
Solution to p. 22 #36

36) From the first four givens, the diagram is relatively easy to draw — the resulting angles shrink down to a minimum size:



Assigning x to the smallest of these angles — $\angle AOF$, — yields us the following measures for the angles:

$m\angle AOF = x$ $\quad$ $m\angle COB = 8x.$

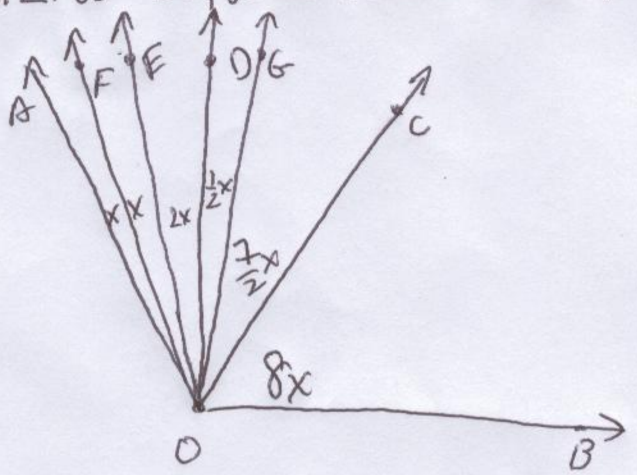
$m\angle FOE = x$

$m\angle EOD = 2x$

$m\angle DOC = 4x$

Since $\overrightarrow{OG}$ bisects $\angle FOC$, each half of the angle should be congruent, so each half measures $\frac{1}{2}(m\angle FOE + m\angle EOD + m\angle DOC) = \frac{1}{2}(x + 2x + 4x) = \frac{1}{2}(7x) = \frac{7}{2}x.$

Since $\frac{7}{2}x > 3x$, we know that the left "half" of $\angle FOC$ should be to the right of $\overrightarrow{OD}$ — that is, $m\angle FOD = 3x$, so $\overrightarrow{OG}$ should be to the right. The diagram becomes:



Since $m\angle FOG = \frac{7}{2}x$, and $m\angle FOG = m\angle FOE + m\angle EOD + m\angle DOG$,

we have: $\frac{7}{2}x = x + 2x + m\angle DOG$

$m\angle DOG = \frac{1}{2}x$, making

$m\angle GOC = 4x - \frac{1}{2}x = \frac{7}{2}x.$

Thus, we have:

$m\angle AOF = x$ $\quad$ $m\angle DOG = \frac{1}{2}x$

$m\angle FOE = x$ $\quad$ $m\angle GOC = \frac{7}{2}x$

$m\angle EOD = 2x$ $\quad$ $m\angle COB = 8x.$

So:

A $m\angle BOF = 120^\circ = 8x + 4x + 2x + x = 15x.$

$120 = 15x$

$x = 8.$

Thus, $m\angle DOE = 2x = 2 \cdot 8 = \boxed{16^\circ}$

B $m\angle COG = 35^\circ = \frac{7}{2}x.$ Thus,

$35 = \frac{7}{2}x$

$\frac{70}{7} = \frac{\cancel{7}x}{\cancel{7}}$

$10 = x.$

$m\angle EOG = m\angle EOD + m\angle DOG$

$= 2x + \frac{1}{2}x$

$= 2 \cdot 10 + \frac{1}{2} \cdot 10 = 20 + 5 = \boxed{25^\circ}$