The Bronx High School of Science
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Course: MGS21(H) – (Honors) Geometry

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Cloning Blues—Congruent Triangle Review

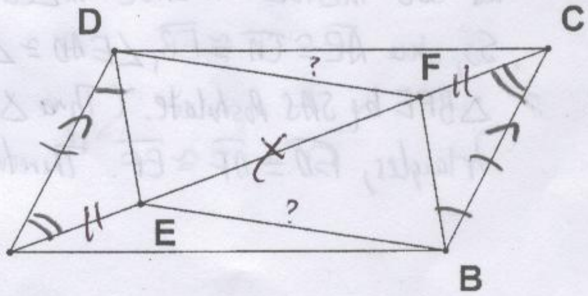
1) **(Easy/Normal)** The angles of an equilateral triangle have measures of $(3x+27)°$, $(2y-4)°$, and $(5z+15)°$. Compute $x+y+z$. Each of the angles of an equilateral triangle are congruent (prove this!), so each angle should measure 60°. Thus:

$3x+27=60$	$2y-4=60$	$5z+15=60$
$3x=33$	$2y=64$	$5z=45$
$x=11.$	$y=32$	$z=9.$

Thus, $x+y+z=11+32+9=\boxed{52.}$

2) Given: $\overline{AD} \cong \overline{BC}$
$\overline{AD} \parallel \overline{BC}$
$\angle ADE \cong \angle CBF$

Prove:
(Normal) $\overline{AB} \parallel \overline{CD}$ (A) **(Hard)** $\overline{DF} \cong \overline{EB}$ (B)

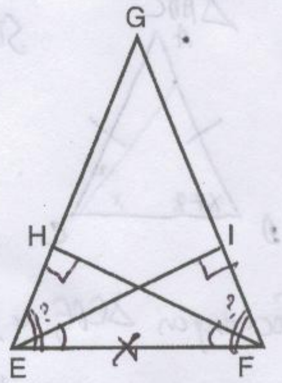


A

Statements	Reasons
1. $\overline{AD} \cong \overline{BC}$	1. Given.
2. $\overline{AD} \parallel \overline{BC}$	2. Given.
3. ABCD is a parallelogram.	3. If a quadrilateral has a pair of parallel, congruent sides, then it is a parallelogram.
4. $\overline{AB} \parallel \overline{CD}$	4. Opposite sides of a parallelogram are parallel.

For B, please refer to the sheet attached to the end of the document.

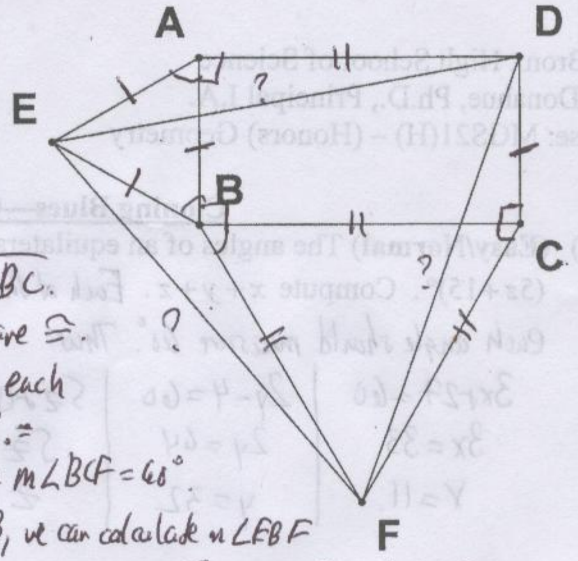
3) **(Normal)**
Given: $\overline{FH}$ and $\overline{EI}$ are altitudes of ΔGEF
$\angle HFE \cong \angle IEF$
Prove: ΔGEF is isosceles



Here, we will focus on the lower two triangles and separate them.

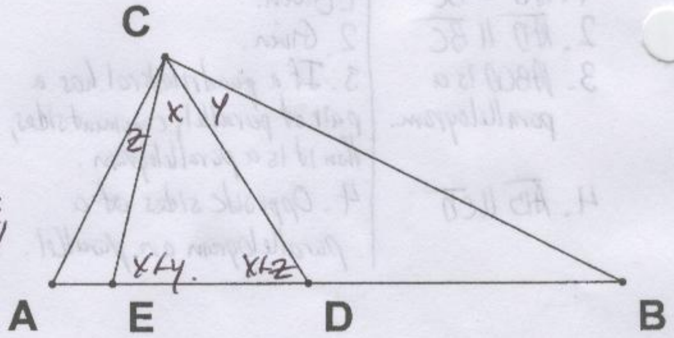
Statements	Reasons
1. $\overline{FH}$ and $\overline{EI}$ are altitudes of ΔGEF.	1. Given.
2. $\overline{FH} \perp \overline{GE}$, $\overline{EI} \perp \overline{GF}$	2. Def. of altitude.
3. $\angle FHE$ and $\angle EIF$ are right angles.	3. Def. of perpendicular lines.
4. $\angle FHE \cong \angle EIF$ (a ≅ a)	4. All right angles are congruent.
5. $\angle HFE \cong \angle IEF$ (a ≅ a)	5. Given.
6. $\overline{EF} \cong \overline{EF}$ (s ≅ s)	6. Reflexive Property
7. $\Delta HFE \cong \Delta IEF$	7. AAS Theorem.
8. $\angle HEF \cong \angle IFE$	8. Corresponding parts of congruent triangles are ≅.
9. ΔGEF is Isosceles.	9. A triangle is isosceles if it has two congruent angles.

4) (**Lunatic**) In the diagram at right, *ABCD* is a rectangle, and *ABE* and *BCF* are both equilateral triangles. Prove that *DEF* is also an equilateral triangle. (*Methinks it's best to do this as a paragraph this time.*)



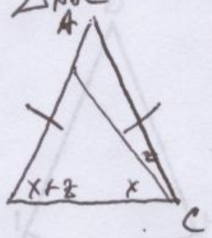
We will aim to show that all of the sides are congruent.
Since the opposite sides of a rectangle are congruent, $\overline{AD} \cong \overline{BC}$.
Also, $\overline{BC} \cong \overline{BF} \cong \overline{CF}$ since all the sides of an equilateral Δ are $\cong$.
By identical reasoning, we have $\overline{AB} \cong \overline{DC} \cong \overline{AE} \cong \overline{EB}$. Since each angle of a rectangle is a right angle, $m\angle DAB = m\angle DCB = 90^\circ$.
Each angle of an equilateral triangle measures 60°, so $m\angle EAB = m\angle BCF = 60^\circ$.
Thus, $m\angle EAD = m\angle DCF = 150^\circ$. If we look around point B, we can calculate $m\angle EBF$ as $360 - m\angle ABE - m\angle ABC - m\angle CBF = 360 - 90 - 60 - 60 = 150^\circ$. Thus, $m\angle EAD \cong \angle DCF \cong \angle EBF$.
So, since $\overline{AE} \cong \overline{CD} \cong \overline{EB}$, $\angle EAD \cong \angle DCF \cong \angle EBF$, and $\overline{AD} \cong \overline{CF} \cong \overline{BF}$, we have $\triangle ADE \cong \triangle CFD \cong \triangle BFE$ by SAS Postulate. (Three Δ's at once!) Thus, as they are all corresponding sides of the triangles, $\overline{ED} \cong \overline{DF} \cong \overline{EF}$. Therefore, by definition, ΔDEF is equilateral.

5) (**Lunatic**) In the diagram at right, ΔABC is drawn such that $\angle ACB$ is a right angle. *D* and *E* are points on $\overline{AB}$ such that $AD = AC$ and $BE = BC$. Compute $m\angle DCE$.



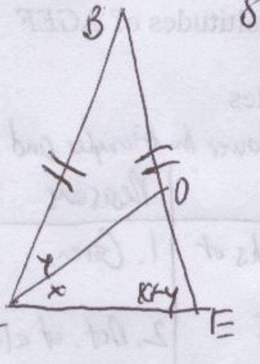
For clarity, we will assign variables. Let $x = m\angle DCE$, $y = m\angle DBC$, and $z = m\angle ACE$. Now, let us focus on two isosceles triangles:

ΔADC



Since $m\angle ACD = x+z$, we know $m\angle ADC = x+z$ as they are base angles.

ΔBCE:



Since $m\angle BCE = x+y$, $m\angle BEC = x+y$ as they are base angles.

Focusing on ΔCDE, we get:

$x+x+y+x+z = 180^\circ$

$2x+x+y+z = 180^\circ$

Wait — $\angle ACB$ is a right angle! Thus,

$z+x+y = x+y+z = 90^\circ$! Thus:

$2x+90 = 180$

$2x = 90 \rightarrow \boxed{x = m\angle DCE = 45^\circ}$

2B) We will aim to show that $\triangle ADF \cong \triangle CBE$, but to do that, we need a little assistance from $\triangle ADE$ and $\triangle CBF$.

Statements	Reasons
1. $\overline{AD} \parallel \overline{BC}$	1. Given.
2. $\angle EDA \cong \angle FCB$ (a ≅ a)	2. If two parallel lines are cut by a transversal, then alternate interior angles are congruent.
3. $\overline{AD} \cong \overline{BC}$ (s ≅ s)	3. Given.
4. $\angle ADE \cong \angle CBF$ (a ≅ a)	4. Given.
5. $\triangle ADE \cong \triangle CBF$	5. ASA Postulate.
~~6. $\overline{AD} \cong \overline{BC}$ (s ≅ s)~~	~~6. Introduced in step 3.~~
6. $\overline{AD} \cong \overline{BC}$ (s ≅ s)	6. Introduced in step 3.
7. $\angle EDA \cong \angle FCB$ (a ≅ a)	7. Introduced in step 2.
8. $\overline{AE} \cong \overline{FC}$	8. Corresponding parts of congruent triangles are congruent.
9. $\overline{EF} \cong \overline{EF}$	9. Reflexive Property.
10. $\overline{AE} + \overline{EF} \cong \overline{EF} + \overline{FC}$ -or- $\overline{AF} \cong \overline{CE}$ (s ≅ s)	10. Addition Postulate.
11. $\triangle ADF \cong \triangle CBE$	11. SAS Postulate.
12. $\overline{DF} \cong \overline{EB}$	12. Corresponding parts of congruent triangles are congruent.